



الجامعة اللبانية  
كلية الإعلام والتوثيق



# Chapter 1 : The Foundation : Logic and proofs

## Lecture 4 : Exercises & Correction

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## Exercise 1

1. Let  $p$  and  $q$  be the propositions:  
     $p$ : You drive over 65 miles per hour.  
     $q$ : You get a speeding ticket.

Write these propositions using  $p$  and  $q$  and logical connectives (including negations).

- a) You do not drive over 65 miles per hour.
- b) You drive over 65 miles per hour, but you do not get a speeding ticket.
- c) You will get a speeding ticket if you drive over 65 miles per hour.
- d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
- e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.
- f) You get a speeding ticket, but you do not drive over 65 miles per hour.
- g) g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

## Solution Exercise 1

(a)  $\neg p$

(b)  $p \wedge \neg q$

(c)  $p \rightarrow q$

(d)  $\neg p \rightarrow \neg q$

(e)  $p \rightarrow q$

(f)  $q \wedge \neg p$

(g)  $q \rightarrow p$

### Exercise 2

Construct a truth table for each of these compound propositions.

**a)**  $p \wedge \neg p$

**b)**  $p \vee \neg p$

**c)**  $(p \vee \neg q) \rightarrow q$

**d)**  $(p \vee q) \rightarrow (p \wedge q)$

**e)**  $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

**f)**  $(p \rightarrow q) \rightarrow (q \rightarrow p)$

## Solution Exercise 2

a)

P	$\sim P$	$P \wedge \sim P$
T	F	F
F	T	F

b)

P	$\sim P$	$P \vee \sim P$
T	F	T
F	T	T

c)

P	Q	$\sim Q$	$(P \vee \sim Q)$	$(P \vee \sim Q) \Rightarrow Q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

d)

P	Q	$(P \vee Q)$	$(P \wedge Q)$	$(P \vee Q) \Rightarrow (P \wedge Q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

e)

P	Q	$\sim P$	$\sim Q$	$(P \Rightarrow Q)$	$(\sim Q \Rightarrow \sim P)$	$(P \Rightarrow Q) \iff (\sim Q \Rightarrow \sim P)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

f)

P	Q	$(P \Rightarrow Q)$	$(Q \Rightarrow P)$	$(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

### Exercise 3

Construct a truth table for each of these compound propositions.

- a)**  $(p \vee q) \rightarrow (p \oplus q)$       **b)**  $(p \oplus q) \rightarrow (p \wedge q)$   
**c)**  $(p \vee q) \oplus (p \wedge q)$       **d)**  $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$   
**e)**  $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$   
**f)**  $(p \oplus q) \rightarrow (p \oplus \neg q)$

## Solution Exercise 3

a)

P	Q	$(P \vee Q)$	$(P \oplus Q)$	$(P \vee Q) \Rightarrow (P \oplus Q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

b)

P	Q	$(P \oplus Q)$	$(P \wedge Q)$	$(P \oplus Q) \Rightarrow (P \wedge Q)$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

c)

P	Q	$(P \vee Q)$	$(P \wedge Q)$	$(P \vee Q) \oplus (P \wedge Q)$
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

d)

P	Q	$\sim P$	$(P \iff Q)$	$(\sim P \iff Q)$	$(P \iff Q) \oplus (\sim P \iff Q)$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

e)

P	Q	R	$\sim P$	$\sim R$	$(P \leftrightarrow Q)$	$(\sim P \leftrightarrow \sim R)$	$(P \leftrightarrow Q) \oplus (\sim P \leftrightarrow \sim R)$
T	T	T	F	F	T	T	F
T	T	F	F	T	T	F	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	T	F	T	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	T	F

f)

P	Q	$\sim Q$	$(P \oplus Q)$	$(P \oplus \sim Q)$	$(P \oplus Q) \Rightarrow (P \oplus \sim Q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

#### Exercise 4

Construct a truth table for each of these compound propositions.

**a)**  $p \rightarrow \neg q$

**b)**  $\neg p \leftrightarrow q$

**c)**  $(p \rightarrow q) \vee (\neg p \rightarrow q)$

**d)**  $(p \rightarrow q) \wedge (\neg p \rightarrow q)$

**e)**  $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$

**f)**  $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

## Solution Exercise 4

a)

P	Q	$\sim Q$	$P \Rightarrow \sim Q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

b)

P	Q	$\sim P$	$\sim P \iff Q$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	F

c)

P	Q	$\sim P$	$(P \Rightarrow Q)$	$(\sim P \Rightarrow Q)$	$(P \Rightarrow Q) \vee (\sim P \Rightarrow Q)$
T	T	F	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	T	F	T

d)

P	Q	$\sim P$	$(P \Rightarrow Q)$	$(\sim P \Rightarrow Q)$	$(P \Rightarrow Q) \wedge (\sim P \Rightarrow Q)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

e)

P	Q	$\sim P$	$(P \iff Q)$	$(\sim P \iff Q)$	$(P \Rightarrow Q) \vee (\sim P \Rightarrow Q)$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

f)

P	Q	$\sim P$	$\sim Q$	$(\sim P \iff \sim Q)$	$(P \iff Q)$	$(\sim P \iff \sim Q) \iff (P \iff Q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

### Exercise 5

Evaluate each of these expressions.

- a)**  $1\ 1000 \wedge (0\ 1011 \vee 1\ 1011)$
- b)**  $(0\ 1111 \wedge 1\ 0101) \vee 0\ 1000$
- c)**  $(0\ 1010 \oplus 1\ 1011) \oplus 0\ 1000$
- d)**  $(1\ 1011 \vee 0\ 1010) \wedge (1\ 0001 \vee 1\ 1011)$

## Solution Exercise 5

a) 1 1000

b) 0 1101

c) 1 1001

d) 1 1011

## Exercise 6

Find the bitwise OR, bitwise AND, and bitwise XOR of each of these pairs of bit strings.

**a)** 101 1110, 010 0001

**b)** 1111 0000, 1010 1010

**c)** 00 0111 0001, 10 0100 1000

**d)** 11 1111 1111, 00 0000 0000

## Solution Exercise 6

a)

OR	AND	XOR
111 1111	000 0000	111 1111

b)

OR	AND	XOR
1111 1010	1010 0000	0101 1010

c)

OR	AND	XOR
10 0111 1001	00 0100 0000	10 0011 1001

d)

OR	AND	XOR
11 1111 1111	00 0000 0000	11 1111 1111

## Exercise 7

Construct a truth table for each of these compound propositions.

**a)**  $(p \vee q) \vee r$

**b)**  $(p \vee q) \wedge r$

**c)**  $(p \wedge q) \vee r$

**d)**  $(p \wedge q) \wedge r$

**e)**  $(p \vee q) \wedge \neg r$

**f)**  $(p \wedge q) \vee \neg r$

## Solution Exercise 7

a)

P	Q	R	$(P \vee Q)$	$(P \vee Q) \vee R$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

c)

P	Q	R	$(P \wedge Q)$	$(P \wedge Q) \vee R$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

b)

P	Q	R	$(P \vee Q)$	$(P \vee Q) \vee R$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

d)

P	Q	R	$(P \wedge Q)$	$(P \wedge Q) \wedge R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

e)

P	Q	R	$\sim R$	$(P \vee Q)$	$(P \vee Q) \wedge \sim R$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

f)

P	Q	R	$\sim R$	$(P \wedge Q)$	$(P \wedge Q) \vee \sim R$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	T	F	T

## Exercise 8

Construct a truth table for  $((p \rightarrow q) \rightarrow r) \rightarrow s$ .

P	Q	R	S	$P \Rightarrow Q$	$(P \Rightarrow Q) \Rightarrow R$	$((P \Rightarrow Q) \Rightarrow R) \Rightarrow S$
T	T	T	T	T	T	T
T	T	T	F	T	T	F
T	T	F	T	T	F	T
T	T	F	F	T	F	T
T	F	T	T	F	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	T	T
F	F	T	F	T	T	F
F	F	F	T	T	F	T
F	F	F	F	T	F	T

Construct a truth table for  $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$ .

P	Q	R	S	$P \leftrightarrow Q$	$R \leftrightarrow S$	$(P \leftrightarrow Q) \leftrightarrow (R \leftrightarrow S)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	T	F	F	T	T	T
T	F	T	T	F	T	F
T	F	T	F	F	F	T
T	F	F	T	F	F	T
T	F	F	F	F	T	F
F	T	T	T	F	T	F
F	T	T	F	F	F	T
F	T	F	T	F	F	T
F	T	F	F	F	T	F
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	F	F
F	F	F	F	T	T	T